

LBYCV2E – THEORY OF STRUCTURES LABORATORY
STAREX NO. 7
Analysis of a Plane Truss

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I. Objectives of the Experiment

- To determine the axial stress and force of the truss member experimentally using strain gauges.
- To discern the axial forces on the truss system and verify the method of joints.
- To assess the joint deflection of a truss system and verify the method of virtual work

II. Procedure

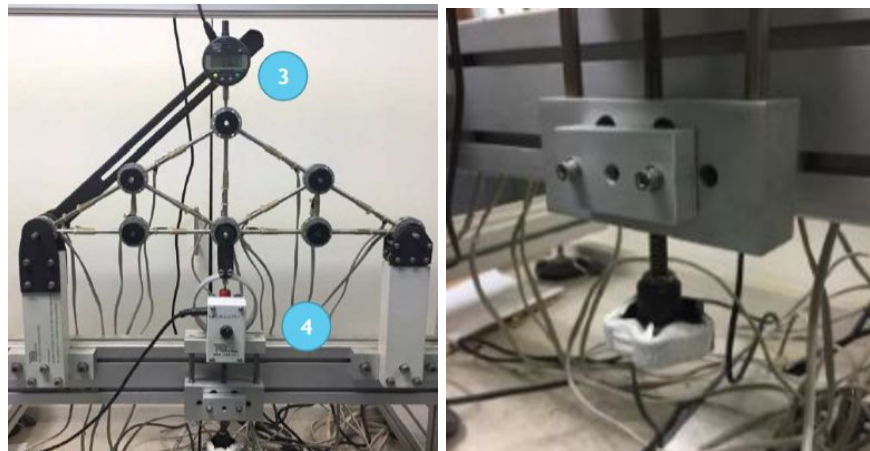


Figure 1. STR-8 Experimental Setup with Load Cell.

In this experiment, the STR-8 apparatus was utilized (as seen in Figure 1). Each group was assigned a specific value of force. With this, the load cell was adjusted until it was loose. The initial load, strain, and displacement transducer reading was set to zero. Subsequently, a load of 150 Newtons, the assigned load was applied by rotating the knob. Then, the record button was clicked in the computer.

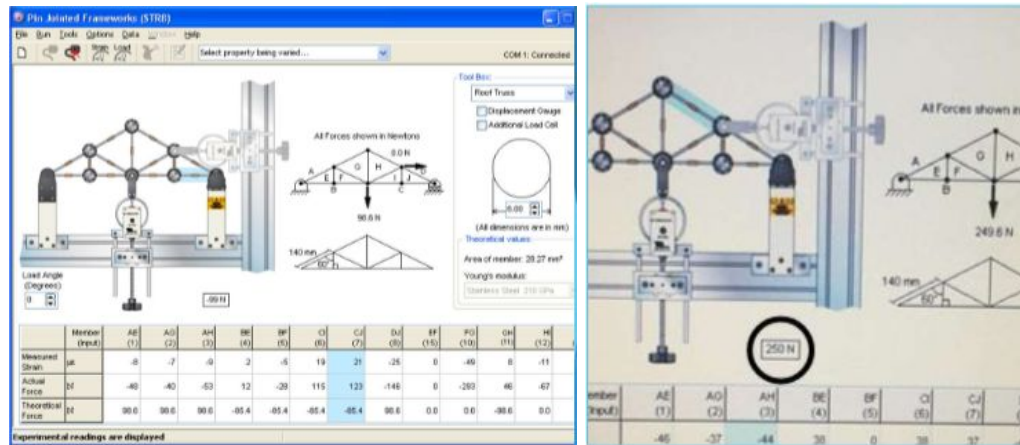


Figure 2. Computer Display

The forces and strains were read from the computer display, and the process was repeated for another trial.

III. Data and Results

Table 1. Measured Strain and Computed Axial Stresses and Forces

Input	1	2	3	4	5	6	7	8	9	10	11	12	13
Bar	AF	FH	GH	AB	BC	CD	DE	EG	BF	CF	CH	CG	DG
e (trial 1)	-29.20	-22.90	-27.30	25.00	0.00	24.30	24.00	-27.70	0.00	1.30	29.00	0.60	0.40
e (trial 2)	-28.20	-22.90	-27.00	24.50	0.00	23.30	23.20	-28.00	0.00	0.60	28.20	0.30	-0.40
e (ave)	-28.70	-22.90	-27.15	24.75	0.00	23.80	23.60	-27.85	0.00	0.95	28.60	0.45	0.00
σ	-6.03	-4.81	-5.70	5.20	0.00	5.00	4.96	-5.85	0.00	0.20	6.01	0.09	0.00
P expt	-176.14	-140.54	-166.62	151.90	0.00	146.06	144.84	-170.92	0.00	5.83	175.52	2.76	0.00
P theory	-150.00	-150.00	-150.0	129.91	129.91	129.91	129.91	-150.00	0.00	0.00	150.00	0.00	0.00

Table 2. Experimental and Theoretical Displacement of Joint H

	Expt displacement (mm)	Theo displacement	Percentage Error
Trial 1	-0.029	-0.029	0.13%
Trial 2	-0.027	-0.029	6.78%
Average	-0.028	-0.029	3.32%

Table 3. Experimental and Theoretical Axial Forces

Bar	AF	FH	GH	AB	BC	CD	DE	EG	BF	CF	CH	CG	DG
P expt	-176.14	-140.54	-166.62	151.90	0.00	146.06	144.84	-170.92	0.00	5.83	175.52	2.76	0.00
P theory	-150.00	-150.00	-150.00	129.91	129.91	129.91	129.91	-150.00	0.00	0.00	150.00	0.00	0.00
% Error	17.42%	6.31%	11.08%	16.92%	100%	12.43%	11.49%	13.95%	0%	100%	14.54%	100%	0%

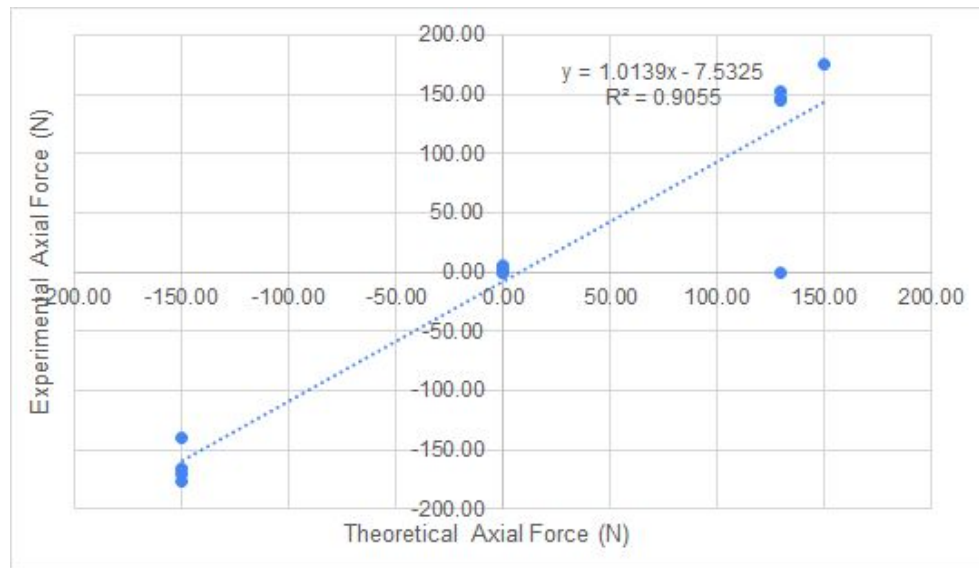


Figure 3. Experimental vs. Theoretical Axial Force

IV. Theory and Calculation

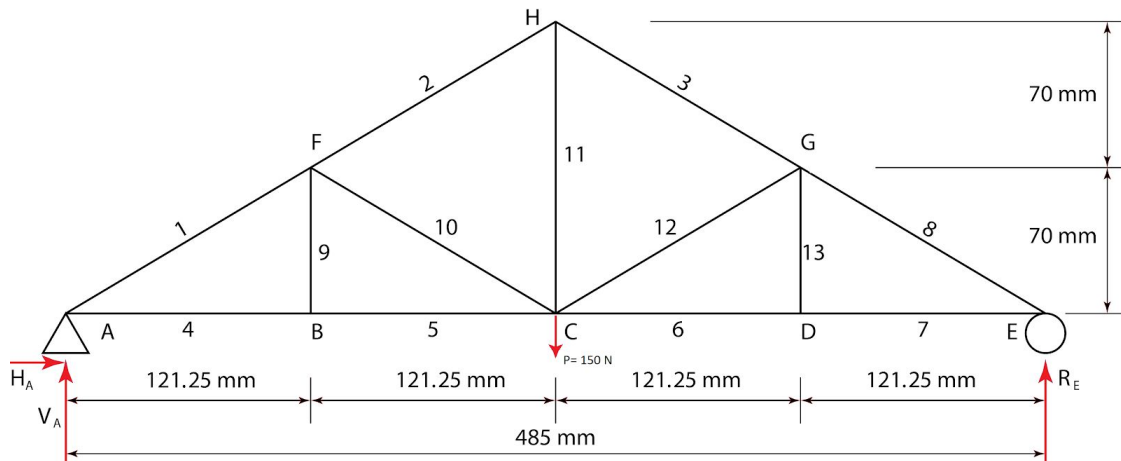


Figure 4. Structural Model of the Truss System with load $P = 150 \text{ N}$ (0.15kN) at Point C.

In this experiment, the given parameters for each member of the truss system made of steel were the following: average bar diameter (6.1 mm), elastic modulus (210 Gpa), and area of a bar (29.22 mm^2). It is also necessary to understand the following concepts which was used cohesively for analyzing the

truss system: p , which signifies the force member along the n th member due to virtual load; P , which represents the force member along the n th member due to load $P = 0.15 \text{ kN}$; A , which symbolizes the cross-sectional area of a member; E which denotes the modulus of elasticity; L which is regarded as the length per member; while, Δ symbolizes the deflection at a certain point.

To identify the deflection at joint H, virtual method which is applicable for multi-link structures may be used to assess the given structural model. In this case, the total work done is proportional to the work of load P at joint C. Provided this, the work of load P is represented by the summation of the integration of each force member multiplied to the change in length of a member due to load P . As such, the total work follows the equation below:

$$(W)(\Delta) = \frac{\sum p P L}{AE} \quad (1)$$

To do this, it is essential to ensure that the reactions were computed using the equilibrium equations. The moments forces were assumed to be in clockwise positive direction, whereas all force members were assumed to be in upward or rightward positive direction:

$$\begin{aligned} \sum M_A &= 0; \\ P(242.5 \text{ mm}) - R_E(485 \text{ mm}) &= 0 \\ R_E &= P/2 \end{aligned} \quad (2)$$

$$\begin{aligned} \sum F_y &= 0; \\ R_E + V_A - P &= 0 \\ V_A &= P/2 \end{aligned} \quad (3)$$

Each member of the truss system was computed through the method of joints. The axial forces were obtained by utilizing two equilibrium equations, particularly the sum of forces along the x or y direction. As a standard procedure, the force members were initially assumed to be in tension such that a negative value connotes that a certain member would be subjected to virtual work method. To obtain the real axial forces (P), Joint A was analyzed by:

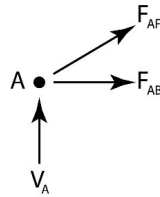


Figure 5. Free Body Diagram at Joint A of the Truss

$$\begin{aligned} \sum F_y &= 0; \\ V_A + F_{AF} \left(\frac{70}{140} \right) &= 0 \\ F_{AF} &= -2V_A \\ F_{AF} &= -P \text{ kN} \end{aligned} \quad (4)$$

$$\begin{aligned}
\sum F_x &= 0; \\
F_{AF} \left(\frac{121.25}{140} \right) + F_{AB} &= 0 \\
F_{AB} &= -F_{AF} \left(\frac{121.25}{140} \right) \\
F_{AB} &= \left(\frac{121.25}{140} \right) (P) \text{ kN} \\
F_{AB} &= \left(\frac{97}{112} \right) (P) \text{ kN}
\end{aligned} \tag{5}$$

With the provided calculations above, substitute equation 5 from the equilibrium equations for joint B:

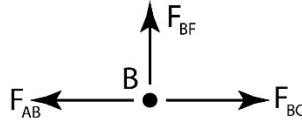


Figure 6. Free Body Diagram at Joint B of the Truss.

$$\begin{aligned}
\sum F_y &= 0; \\
F_{BF} &= 0
\end{aligned} \tag{6}$$

$$\begin{aligned}
\sum F_x &= 0; \\
F_{BC} - F_{AB} &= 0 \\
F_{BC} &= F_{AB} \\
F_{BC} &= \left(\frac{121.25}{140} \right) (P) \text{ kN} \\
F_{BC} &= \left(\frac{97}{112} \right) (P) \text{ kN}
\end{aligned} \tag{7}$$

Substituting equation 6 from the equilibrium equations for joint F:

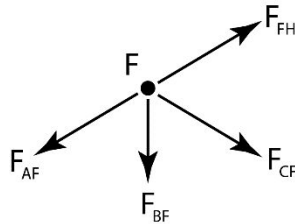


Figure 7. Free Body Diagram at Joint F of the Truss.

$$\begin{aligned}
\sum F_y &= 0; \\
-F_{AF} \left(\frac{70}{140} \right) - F_{CF} \left(\frac{70}{140} \right) + F_{FH} \left(\frac{70}{140} \right) - F_{BF} &= 0 \\
-F_{AF} - F_{CF} + F_{FH} &= 0 \\
F_{FH} &= F_{CF} + F_{AF} \\
F_{FH} &= F_{CF} - P
\end{aligned} \tag{8}$$

$$\begin{aligned}
\sum F_x &= 0; \\
-F_{AF} \left(\frac{121.25}{140} \right) + F_{CF} \left(\frac{121.25}{140} \right) + F_{FH} \left(\frac{121.25}{140} \right) &= 0 \\
-F_{AF} + F_{CF} + F_{FH} &= 0
\end{aligned}$$

$$\begin{aligned}
F_{FH} &= -F_{CF} + F_{AF} \\
F_{FH} &= -F_{CF} - P
\end{aligned}
\tag{9}$$

From the given equations, substitute equation 8 to 9 to obtain the actual value for F_{CF} :

$$\begin{aligned}
F_{CF} - P &= -F_{CF} - P \\
2F_{CF} &= 0 \\
F_{CF} &= 0
\end{aligned}
\tag{10}$$

From the given equations, substitute equation 10 to 8 to obtain the actual value for F_{FH} :

$$\begin{aligned}
F_{FH} &= 0 - P \\
F_{FH} &= -P \text{ kN}
\end{aligned}
\tag{11}$$

Substituting equation 10 and 11 from the equilibrium equations for joint C:

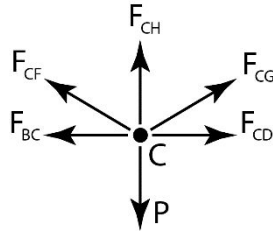


Figure 8. Free Body Diagram at Joint C of the Truss.

$$\begin{aligned}
\sum F_y &= 0; \\
-P + F_{CH} + F_{CG} \left(\frac{70}{140} \right) + F_{CF} \left(\frac{70}{140} \right) &= 0 \\
F_{CH} &= P \text{ kN}
\end{aligned}
\tag{12}$$

Analyzing the truss system, the structure appeared to be symmetrical. With this, the axial forces on the left hand side as it is cut into half was proportional to the corresponding force members on the right hand side:

$$F_{AB} = F_{DE} \tag{13}$$

$$F_{BC} = F_{CD} \tag{14}$$

$$F_{AF} = F_{GE} \tag{15}$$

$$F_{BF} = F_{DG} \tag{16}$$

$$F_{CF} = F_{CG} \tag{17}$$

$$F_{FH} = F_{GH} \tag{18}$$

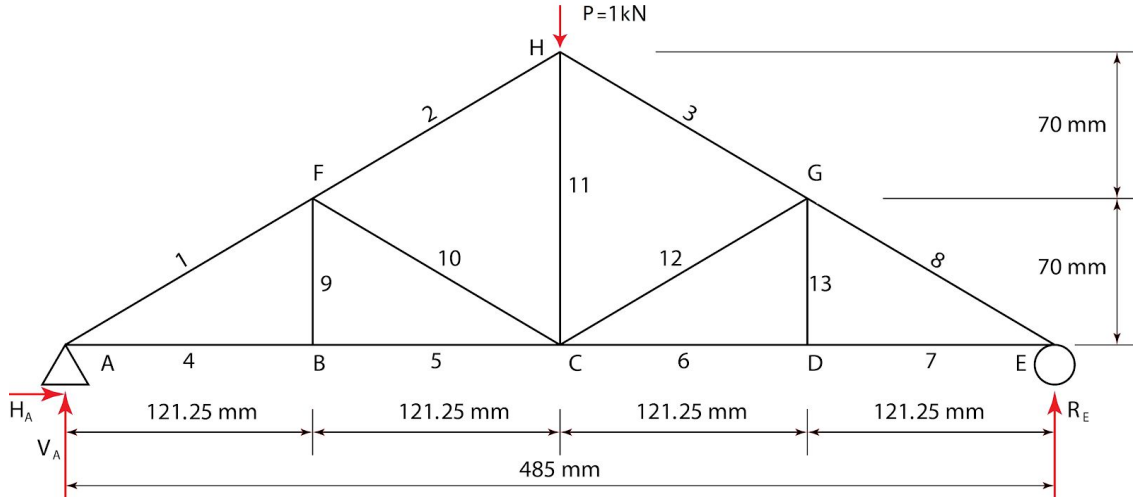


Figure 9. Virtual Structural Model of the Truss System with a Virtual Load $P = 1 \text{ kN}$ at joint H.

Since the actual axial forces were calculated, the virtual load may be obtained using the equilibrium equations. A virtual structural model assessment may be performed by assigning a unit load which is 1 kN at the point where a vertical or horizontal displacement is preferred. For this particular experiment, a vertical displacement at joint H is to be analyzed. The process of acquiring the virtual forces in each member resembles that of acquiring the actual axial forces except that external loads were disregarded as a virtual load P of 1 kN was assigned at joint H.

$$\begin{aligned}\sum M_A &= 0; \\ 1 (242.5 \text{ mm}) - R_E (485 \text{ mm}) &= 0 \\ R_E &= 0.5 \text{ kN}\end{aligned}\tag{19}$$

$$\begin{aligned}\sum F_y &= 0; \\ R_E + V_A - P &= 0 \\ 0.5 + V_A - 1 &= 0 \\ V_A &= 0.5 \text{ kN}\end{aligned}\tag{20}$$

From joint A (Figure 5), the equilibrium equations were:

$$\begin{aligned}\sum F_y &= 0; \\ V_A + F_{AF} \left(\frac{70}{140} \right) &= 0 \\ F_{AF} &= -2V_A \\ F_{AF} &= -1 \text{ kN}\end{aligned}\tag{21}$$

$$\begin{aligned}\sum F_x &= 0; \\ F_{AF} \left(\frac{121.25}{140} \right) + F_{AB} &= 0 \\ F_{AB} &= -F_{AF} \left(\frac{121.25}{140} \right)\end{aligned}$$

$$F_{AB} = \frac{97}{112} \text{ kN} \quad (22)$$

From joint B (Figure 6), substitute equation 22 to 24 from the equilibrium equations,

$$\begin{aligned} \sum F_y &= 0; \\ F_{BF} &= 0 \end{aligned} \quad (23)$$

$$\begin{aligned} \sum F_x &= 0; \\ F_{BC} - F_{AB} &= 0 \\ F_{BC} &= F_{AB} \\ F_{BC} &= \frac{97}{112} \text{ kN} \end{aligned} \quad (24)$$

From joint F (Figure 7), substitute equation 21 to 25 and 26 from the equilibrium equations:

$$\begin{aligned} \sum F_y &= 0; \\ -F_{AF} \left(\frac{70}{140} \right) - F_{CF} \left(\frac{70}{140} \right) + F_{FH} \left(\frac{70}{140} \right) - F_{BF} &= 0 \\ -F_{AF} - F_{CF} + F_{FH} &= 0 \\ F_{FH} &= F_{CF} + F_{AF} \\ F_{FH} &= F_{CF} - 1 \end{aligned} \quad (25)$$

$$\begin{aligned} \sum F_x &= 0; \\ -F_{AF} \left(\frac{121.25}{140} \right) + F_{CF} \left(\frac{121.25}{140} \right) + F_{FH} \left(\frac{121.25}{140} \right) &= 0 \\ -F_{AF} + F_{CF} + F_{FH} &= 0 \\ F_{FH} &= -F_{CF} + F_{AF} \\ F_{FH} &= -F_{CF} - 1 \end{aligned} \quad (26)$$

From the given equations, substitute equation 25 to 26 to obtain the value for F_{CF} :

$$\begin{aligned} F_{CF} - P &= -F_{CF} - 1 \\ 2F_{CF} &= 0 \\ F_{CF} &= 0 \end{aligned} \quad (27)$$

From the given equations, substitute equation 27 to 26 to obtain the value for F_{FH} :

$$\begin{aligned} F_{FH} &= 0 - 1 \\ F_{FH} &= -1 \text{ kN} \end{aligned} \quad (28)$$

From joint C (Figure 8), substitute equation 21 and 22 from the equilibrium equations:

$$\sum F_y = 0;$$

$$-1 + F_{CH} + F_{CG} \left(\frac{70}{140} \right) + F_{CF} \left(\frac{70}{140} \right) = 0$$

$$F_{CH} = 1 \text{ kN} \quad (29)$$

Table 4. Virtual Work Method for Vertical Displacement Assigned at Joint H.

Bar	Length (mm)	P (kN)	p (kN)	PpL
AF	140	-0.15	-1	21
FH	140	-0.15	-1	21
GH	140	-0.15	-1	21
AB	121.25	0.13	1.15	18.1875
BC	121.25	0.13	1.15	18.1875
CD	121.25	0.13	1.15	18.1875
DE	121.25	0.13	1.15	18.1875
EG	140	-0.15	-1	21
BF	70	0	0	0
CF	140	0	0	0
CH	140	0.15	1	21
CG	140	0	0	0
DG	70	0	0	0
			TOTAL:	177.75
			ΔH	0.029

For the virtual structural model, the axial forces on the right hand side were computed from equations 13 to 18. The force members (as seen in Table 4) were tabulated to determine the necessary values such that:

$$W = \frac{\sum P_i P_i L}{AE}$$

$$(W)(\Delta H) = \frac{177.75}{(29.22 \text{ mm}^2)(210 \text{ 000 000 } \frac{\text{N}}{\text{m}^2}) \left(\frac{1 \text{ m}}{1000 \text{ mm}} \right)^2 \left(\frac{1 \text{ kN}}{1000 \text{ N}} \right)}$$

$$\Delta H = 0.02896744 \text{ mm} \quad (30)$$

$$\sigma = \epsilon E$$

The experimental axial stress σ may be computed by simply using the aforementioned formula such that the strain of a specific member must be multiplied with the modulus of elasticity.

$$\sigma = \frac{P}{A}$$

while, the experimental axial force each member may be computed by deriving the respective formulas wherein:

$$\begin{aligned}\epsilon E &= \frac{P}{A} \\ P &= \epsilon EA\end{aligned}\tag{31}$$

V. Data Analysis and Conclusions

In this experiment, the deflection of a symmetrical truss and the axial forces within the beams were analyzed. The proponents of the experiments were able to analyze the theoretical and experimental values of the assigned load of 150 Newtons as well as the vertical displacement at joint H. The values for the axial forces and the deflection were obtained from the STR-8 apparatus and was compared to the theoretical value. To obtain the theoretical deflection, virtual work method and method of joints were employed.

Table 1 shows the strains obtained from each member. This was done in two (2) trials then the average strain was calculated. The axial stress σ of each member was computed by multiplying the strain of the specific member to the modulus of elasticity. The theoretical axial forces were obtained by using the method of joints as stated in the theory above. As for the experimental axial forces, Eq. 31 was used to obtain such values. Theoretically, it would be observed that members BF, CF, CG, and DG are zero force members. However in the experiment, members CF and CG obtained an axial force of 5.83 and 2.76, respectively. Apart from this, member BC should have an axial force of 129.91 but obtained a zero force in the experiment. Furthermore, Table 3 exhibits the obtained experimental and theoretical axial forces on each member. The percentage errors between the aforementioned variables would be observed to be significantly high especially between the axial forces of members BC, CF, CG wherein a 100% error was calculated. Despite this, when the experimental axial forces are plotted against their corresponding theoretical axial forces (as shown in Figure 1), an R^2 value of 0.9055 was obtained. This indicates that the two variables have a strong correlation and that their values align with each other.

Table 2 shows the comparison of the experimental and theoretical axial forces for the 13 bar segments. The highest positive and negative value for experimental force for all bars is -176.14 N in bar AF and 175.52 N, respectively. Bar BF and bar DG both got an experimental and theoretical force equal to zero, resulting in a zero percentage error as well. The highest percentage error obtained is 17.42% at bar AF and the lowest percentage error is 6.31% at bar FH. Figure 1 shows a plot of the experimental with respect to the theoretical axial force. From the graph, we obtained a positive linear relationship with a trendline equation of $y = 1.0139x - 7.5325$. The coefficient of determination is 0.9055. This implies that the data points are strongly correlated to one another and the regression line predicted the data points well.

The initial value for the theoretical vertical displacement at joint H obtained a positive value as compared to the expected negative value of its experimental displacement. This may be due to an error in the sign convention committed by the proponents of the experiment while computing, setting the equipment, or the data being read by the computer may not be accurate as it may not have been calibrated properly. Specifically, the contributors of errors for the experiment are the external factors and human errors. The external errors can be the change of the environment where the test is being held on. The change of environment can affect the performance of the equipment and the material that is being used for this. The humidity and the temperature can change once the environment is not done in a controlled area. This is a reason why experiments must be conducted in a controlled environment in order to get the results that the researchers want to. When an experiment is being done the researchers must always be aware of what is happening to the experiment. The researchers must also double check the process that they are doing to know if all the process is done correctly. Using the whole value for the computation can also be a big factor. Not using the whole value can result in having big discrepancies in the expected outcome by the researchers.